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THE INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS.*

By **DAVID EUGENE SMITH**, President of the American Commission.

So much has been said and written of late concerning the work of the International Commission on the Teaching of Mathematics, and so closely connected are several members of this Association and this Society with the movement, that I find myself quite at a loss in attempting to impart any new information as to the inception of the work and the general purposes in view. Nevertheless a brief resumé of the organization of the Commission may not be out of place, to be followed by a statement of the problems that particularly confront us here in America.

At the Fourth International Congress of Mathematicians, held at Rome in 1908, the following resolution was adopted: "The Congress recognizing the importance of a comparative examination of the methods and plans of study of the instruction in mathematics in the secondary schools of the different nations, empowers Messrs. Klein, Greenhill, and Fehr to form an International Commission, to study these questions and present a general report to the next Congress."

It was by no means a simple matter to effect this preliminary organization. The general idea was popular — if anything, too popular, and the plan seemed for a time on the point of failing because of this very fact. The difficulty lay in the selection of the organizing committee, the naming of which on the basis of the countries represented would have resulted in an unwieldy body, and one that would naturally have been selected too hurriedly for the best results. In the closing moments of the session of of Section IV, however, there seemed to be a general consensus of opinion that three men were especially fitted to undertake the work of organization,— Professor Klein, because of his high mathematical attainments and his great interest in education; Professor Sir George Greenhill, because of his eminence in the field of applied mathematics and because he represented the country in which the next Congress is to be held; and Professor H. Fehr, because of his position as editor of *L'Enseignement Mathématique.* It was expected that these gentlemen would constitute, in the order named, the President, Vice President, and General Secretary of the International Commission which they were empowered to organize, and this expectation has since been realized. Their selection was unanimously approved by the Congress in general session, together with the resolution already mentioned.

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*An address delivered before Section A of the American Association for the Advancement of Science, and The American Mathemathical Society, in joint session, at Boston, December 29, 1909.

These gentlemen set about, in due time, to select representatives from the various countries usually participating in these congresses, consulting with members of learned societies, I am told, and during the year now closing the organization has practically been effected. It would not be appropriate at this time to give the complete list of commissioners chosen, but the names of a few who are acting in this capacity, or are closely connected with the movement as members of important national committees, or otherwise, will serve to show that the work is to be performed in no perfunctory fashion.

Among those who have been chosen to carry on the investigation in Germany are Klein, Staeckel, Gutzmer, Treutlein, Schotten, and Shimmack; in France, Saint-Germain, Bourlet, Appell, Borel, Tannery, André, and Laisant; in Italy, Castelnuovo, Enriques, and Scorza; in Austria, Czuber, Suppantschitsch, and Wirtinger; in Hungary, Beke, Rados, Rátz, and Goldziher; in Great Britain, Greenhill, Godfrey, Fletcher; in Russia, Sonin, Kojalovitch, Vogt; in Denmark, Heegaand, Juel, Trier; in Switzerland, Fehr, Graf, Moser,—all these being men that stand among the leaders in their lines. In the United States, there have accepted positions upon the Advisory Council the president and all of the living ex-presidents of the American Mathematical Society, the president of the American Federation of Teachers of the Mathematical and Natural Sciences, the presidents of Harvard University, Columbia University, and the University of Chicago, and the United States Commissioner of Education. In our country are encluded in the membership of the various committees and sub-committees many of our best known mathematicians and teachers of mathematics, and here, as abroad, the best of spirit has been shown towards the work.

The question that naturally arises in considering such a movement is the very ancient one, *Cui bono?* It is a perfectly proper question and it serves the more clearly to fix in the minds of those charged with responsibility, the definite purpose of the investigation.

Before attempting a brief reply, it may be proper to state what is not the purpose, to the end that the real aims may stand forth the more clearly. It is not the purpose of the Commission, nor of any of its members, so far as appears, to pose as a body of reformers. Nothing has yet developed to show that anyone connected with the movement believes that mathematics is more poorly taught than any other subject, or is in more need of reform. No revolution has been advocated, nor has any one expressed any desire to have the teaching of mathematics remain stagnant. All articles that have thus far been written in connection with the work show that the members of the Commission desire the same substantial progress in teaching mathematics that is generally found in all other subjects, and no one has taken the attitude of opposition to all that exists, nor has anyone appeared with the dictum that all that is, is bad and that the new and untried is always good. This general attitude of the Commission, if we may judge by what has thus far been done, will argue for lack of progress with some earnest workers, but it is probable that it will meet with the approval of the substantial leaders in mathematics and in education today.

In the second place, it is evidently not the purpose of the Commission to make any direct attempt at uniformity in the various countries. The very manner of conducting the investigation shows that each country has gone to work at the problem in the way that seems to it best, without the slightest regard to the methods employed in other countries; and this independence, this recognition of the peculiar needs, aims, and means at the disposal of the various countries, will undoubtedly continue to be seen as the work progresses. It is not, therefore, proposed to revolutionize the teaching of mathematics, nor is it proposed to attempt to bring all nations to one point of view as to schools, as to teaching, as to subjects taught, or as to curricula.

From the time of the Greek mathematicians until now, it has been recognized that a negative statement is never satisfactory, and so it is proper to proceed to a positive statement of some of the reasons for this word-wide investigation.

The prime reason is unquestionably to be found in the value of an interchange of views and experiences among nations. Why should the American Association for the Advancement of Science, or the American Mathematical Society, or the American Federation of Teachers of the Mathematical and Natural Sciences exist? Is it not enough that we have the faculty meetings, and local reading circles, and even state organizations? Have we not journals and reports and text-books and treatises to keep us mentally alert? Then why do we need such national gatherings as the present one, and why should we give up our well-earned holidays to attend them? These questions seem puerile when applied to our present gathering, for each of us is conscious that he gains much inspiration from such a meeting, and that he returns to his work with a broader horizon because he has been conducted to other points of view. He sees the national problem instead of his narrow local one, and his work is the stronger because of this fact. You cannot weigh this value, nor is it to be measured in volume or in depth, for it is incommensurable with any units of the laboratory or of trade,—but our sixth sense knows it, and it is even more real than it would be if expressed in the mere figures of a scale.

And if the village point of view is narrow, and even that of the city, and of the state, and of this section or that of a country like ours, so too is the national point of view narrow as compared with that of the world of which it must be, whether it will or no, a part. This, then, is the great *raison d'être* of the Commission,—to act as a clearing house, to let each part of the world see what the other part is doing in the teaching of mathematics. Uniformity is no more desirable nor attainable than uniformity in dress, in government, or in speech. The movement will tend in this direction, just as all intercourse tends to uniformity in these other lines, but centuries will elapse before any of these results will be reached, if unfortunately they are ever attained.

Such a movement, looking to telling the rest of the world what any specific nation is doing in the teaching of mathematics, carries with it the necessity, on the part of that nation, of taking stock of its own results, and of indulging in a very healthy form of introspection. It is most interesting to those of us who are intimately connected with the investigation, to see how much we all have to learn of the courses of study, the amount that can be well done in a year, and the topics that are treated in the various parts of our country. I will illustrate by two simple but typical questions: (1) What are the aims, and consequently what should be the nature of the work, in mathematics, in the large city commercial high schools of this country? (2) What are the requirements in mathematics for the doctor's degree in say three universities belonging to the Association of American Universities? It would seem that these questions ought to be easily answered by several persons in an audience like this, but we find difficulty in discovering anyone to answer them, in a much larger circle than the one here represented. A second large reason, then, for this investigation is that we may know not only what the world at large is doing, but also our own aims and curricula and efforts at improvement.

But more specfically, what is it that we propose to investigate?—that the rest of the world wishes to know? that we ourselves need also to inquire into?

In the first place, it was at once seen that the limitation to secondary schools was meaningless. The term signifies one thing in America, but something radically different in Germany, and something different still in other countries. Even with us there is no well-marked boundary between the elementary school with six, seven, eight, or nine years, and the secondary school with six, five, or four years, or even less. It was therefore early decided to extend the investigation so that it might include the entire field of mathematics required for any purpose, from the kindergarten through the college, the technical school, and the professional school. Besides this, since the investigation includes the preparation of teachers, it must also include a statement of the general nature of the graduate work in mathematics in the universities, and in particular of the requirements for the various advanced degrees in universities of recognized standing.

With the general scope of the work thus set forth, I pass to a consideration of a few of the large topics to be investigated in this country.

The first of these topics is the mathematics of the general elementary school, where a few years ago the obsolete business problem and the remains of the ancient theory of numbers were looked upon as of paramount importance, but where now the needs of modern daily life make up the bulk of the work. The change here has been greater than in any other part of our American education in mathematics, save only in the graduate university work. To report upon this change, and to set forth the present status of the work, for the various types of schools, is a labor of no little difficulty, and the world at large will look upon it as one of our most important contributions.

There is next considered certain special kinds of elementary schools, of which we hear and know little because they are still so new. The mathematics of the trade school is not that of the public school, nor is that of the large corporation industrial schools, now springing up in various parts of the country, like that of the trade school. We have not yet differentiated in this field as they have, for example, in Switzerland, and perhaps we shall never need to do so, but it is certain that these schools are with us, and that others are coming, and that their needs in mathematics, and their ways of meeting these needs, will be of interest to all who are engaged in the furthering of education.

One of the largest problems that confronts us is the study of the public general secondary schools of the country, partly because it is here that mathematics, beyond mere computation, may be said to begin, and partly because there is more criticism of this work than of any other save in the first two years of college. Fortunately we have in Mr. Evans of this city an organizer of proved ability, and with him we have a committee of very earnest workers. Anyone who is at all familiar with these public schools knows that there is much divergence in the courses in mathematics in different parts of the country, and in schools intended for boys, for girls, or for both. It is also apparent that the new movement towards a six-year elementary school, followed by a six-year high school, will considerably modify the mathematics of the latter. With six years in the high school, the present mathematics will begin earlier, it may possibly be taken more leisurely, and eventually it may well lead into higher lines of work. The problem, therefore, assumes an interest that a few years ago would not have been expected. Add to this the numerous efforts made at present to frame new syllabi in subjects like algebra and geometry, and it is evident that the Committee has a labor of no small extent to perform.

Then there is another problem that is quite different, namely, that of the private secondary school, whether for boys alone, for girls alone, or for both together. These are generally fitting schools for college. Instead of a small per cent. going to college, as in the public schools, the rest going into business, here these per cents are reversed, and the nature of the work in mathematics is, or may be, very different. As a matter of fact, it is different, and more different than one would at first expect.

An important question, and one that is by no means settled, is that which relates to the mathematics in the normal schools, public and private. Upon this depends the equipment of the teachers who are to instruct the new generation of elementary pupils in the early principles of mathematics, largely determining whether or not they will care for the subject in later years. Of late there has been a tendency to weaken the mathemathics in these schools, and it is hoped that this report will determine the needs and perhaps the future policy of such institutions in this important field.

There is arising in our country at present a type of secondary school that is well known on the continent of Europe but that has yet to make its way with us. This is the technical secondary school. We have, for example, even in the present year a notable development of the agricultural high school, of the commercial high school, and of the general technical or manual training high school. What shall the mathematics be in these institutions? Shall it be Euclid revised and algebra through quadratics, as in the ordinary high school? Or shall it be mathematics correlating directly with the peculiar work of the type of school in question?

Related to these schools is the rapidly increasing number of evening technical schools, of private correspondence schools, of schools for licensed accountants, and of industrial schools for negroes and Indians. Some of these institutions are teaching mathematics even through applied calculus, and are doing it with a spirit that would put the average college class to shame, and it will help us all to know their work more fully.

One of the most important problems that we have relates to the examination question. We commonly look at England as examination-ridden, and in the East we often hear it said that the West and Middle West teach mathematics so loosely that they dare not submit their pupils to the Eastern tests. Here, therefore, are these types,—the English, the extreme of the examination system; the West, the extreme of the non-examination system; and the East occupying a middle ground. It is well that we endeavor fairly to weigh the advantages of these various plans, to consider the nature of the promotion in the elementary grades, to investigate the results of the several plans for admission to college, and to inform ourselves as to the examinations in mathematics set by the state and local authorities for those desiring to teach in the public schools. If we do not, as a result of all this work, attain to uniformity, at least we shall be more tolerant of the plans of others, if not of our own.

America has of late branched out geographically for good or for ill, and it is expected that we shall hear, as part of this report, of the work being done in mathemathics in the Philippines, in Hawaii, in Porto Rico, and in Alaska. Nor will this be merely a discussion of the elementary schools, for Hawaii has more than one good college, and Manila has a university so old that most of ours seem young in comparison.

We have in this country certain influences that tend to improve the work of the teacher that will be of interest to other countries, and about which we ourselves may profitably be better informed. Our scientific societies and periodicals are stimuli to our scientific activity, and are characterized by a generosity of spirit that will be the envy of several countries. The nature of these organizations and publications, including the American Association for the Advancement of Sciences, and our various mathematical journals, will be the subject of one of our important reports. We are favored, too, in having so many associations of teachers of mathematics, and to them we owe most of the syllabi in algebra and geometry that are now appearing. The teachers' institute, as conducted in our various states, seems to be indigenous to our country, and while it has not in general done much for mathematics, it has, in the high school sections, often proved of service. These and similar questions will be discussed by a committee appointed for this special purpose.

There has grown up in this country a line of technological work which we have reason to regard with much satisfaction,—that of collegiate grade, represented by institutions like the Massachusetts Institute of Technology. Such schools are not unique with this country; indeed, we borrowed the idea from Germany, Switzerland, France, and Belgium. But they have steadily increased in number, in dignity, and in their ability to offer strong courses in engineering mathematics. But what this work is, and how it differs in the technological schools from the work in the technological departments of colleges and universities, are matters upon which we should all welcome exact information.

Allied to this problem is that of the mathematics taught in other professional schools, such as those for the training of army and navy officers, and particularly in the higher institutions for the training of teachers that are beginning to be established in this country. The question of how we proceed to train a teacher of mathematics for the secondary school and college, in the highest type of college for this purpose, will be of interest abroad and not without its value at home.

From the educational standpoint the mathematical work of the college is commonly felt, in this country, to be the least progressive. Perhaps it should be so, but at any rate it will be of service to us to know exactly what is expected of the freshman year, and of the sophomore year, in an average American college. Whatever exact information we can get as to the aims, the general plan of presentation, and the results, of these two years, will be stimulating in all institutions of this type. Perhaps as important a question as we have to consider is that relating to the nature of the mathematics in the women's colleges, compared with that in the colleges for men. Is it the same? Is the ability the same? Such are some of the questions that the world of mathematics would like to have scientifically considered.

Perhaps the most interesting question of all, from the standpoint of mathematics, is that relating to the graduate work done in the universities of this country,—that is, in the few institutions of high grade that are generally accepted as prepared for genuine work of this character. One of our committees has this problem in hand, one of its sub-committees having the question of the courses of instruction, another of the preparation for research and the doctor's degree, and another of the preparation of instructors for colleges and universities. The report of this committee may well be expected to set a standard of excellence that shall be recognized for many years to come, in this country.

And finally, there are certain general questions that need to be considered, particularly for the information of other countries. One of these has to do with a schematic survey of American educational institutions, their sequence and inter-relations. The fact that this particular topic is under the direction of a sub-committee headed by the Commissioner of Education of the State of Massachusetts, is a guarantee that the work will be well done. The second of these general topics has to do with the scope and arrangement of the mathematical curriculum as a whole, as distinguished from the arrangement of limited portions as considered by various other committees.

In America, as in some other countries, the work is carried on by means of committees. Certain other countries are conducting the investigation through individuals. In this country we have fifteen committees, sixty-one sub-committees, and a total of about two hundred and seventy members. These committees and sub-committees are working out their problems in their own way, some by means of elaborate questionnaires, others through associations, others from statistical reports, and others through investigations previously made by their members. It is hoped that a considerable number of the sub-committee reports will be ready in February, 1910, and that all of the committee reports will be ready the following May. Some extensions must be allowed, but it is hoped that these will not delay the ultimate stages of the work. The American report must be ready a year hence, and the great labor that must be expended upon it leads to the hope that all who receive questionnaires will respond heartily and speedily to the end that the burden may not be made any heavier.

Such is the general purpose and such is the general plan of this work. It imposes upon all of us who are connected with it a very heavy burden. But the work must be done for the reputation of the country and for the sake of its effect both here and abroad. It is to the sympathetic interest of societies like those that I address that we look for our chief encouragement and assistance, and it is upon teachers and mathematicians like yourselves that we have called to fill the various committees and to work with us, *con amore* as all of us must, to make the labor result in success and be of value to ourselves and to our confères in other parts of the world.